WL-TR-91-3101

IDENTIFICATION OF DYNAMIC STRUCTURES WITH SPECIAL ATTENTION TO DAMPING MATRIX REPRESENTATION

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January 1992

Final Report for Period May 1991 - September 1991

Approved for public release; distribution unlimited.

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REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

1. AGENCY USE ONLY (Leave blank)	2. REPORT DATE	3. REPORT TYPE AND DATES COVERED
	13 January 1992	Final 8 May 91-25 Sep.91

4. TITLE AND SUBTITLE
Identification of Dynamic Structures With Special Attention to Damping Matrix

5. FUNDING NUMBERS
F33615-90-C-3211
PE-62201F
PR-2401
TA-02
WU-94

6. AUTHOR(S)
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7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
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8. PERFORMING ORGANIZATION REPORT NUMBER
ASIAC-TR-91-1

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)
Capt. Stephen Rasmussen
Flight Dynamics Directorate (WL/FIBR)
Wright-Laboratory
Wright-Patterson AFB, OH 45433-6553

10. SPONSORING/MONITORING AGENCY REPORT NUMBER
WL-TR-91-3101

11. SUPPLEMENTARY NOTES

12a. DISTRIBUTION/AVAILABILITY STATEMENT
Approved for public release;
distribution unlimited

12b. DISTRIBUTION CODE

13. ABSTRACT *(Maximum 200 words)*

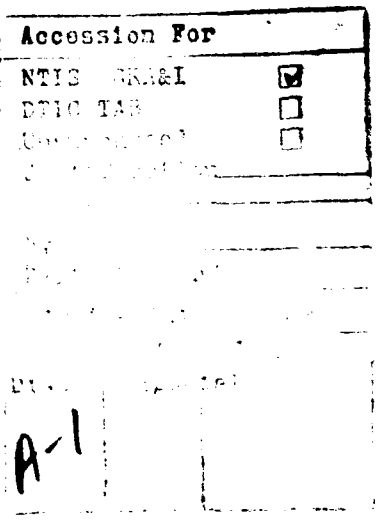
In finite element modeling, damping parameters, structural characteristics of joints, and boundary conditions are very difficult to estimate, causing discrepancy between analytic results and actual measured behavior. Various methods of structural identification were studied and compared in terms of their basic assumptions and advantages. A nonlinear optimization based method was thought to be promising for the identification of structures with highly uncertain structural parameters. An algorithm was developed to construct the damping matrix from experimental data. A computer program was developed for verification of this method. The method appeared to work well for lightly, viscously damped structures.

14. SUBJECT TERMS	15. NUMBER OF PAGES
Finite Element Analysis, Nonlinear Optimization, Damping, Dynamics, Response, Proportional Damping, Viscous Damping	28
	16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
Unclassified	Unclassified	Unclassified	UL

FOREWORD

This report was prepared by the Aerospace Structures Information and Analysis Center (ASIAC), which is operated by CSA Engineering, Inc. under contract number F33615-90-C-3211 for the Flight Dynamics Directorate, Wright-Patterson Air Force Base, Ohio. The report presents the work performed under ASIAC Task No. T-09. This effort was sponsored by the Analysis and Optimization Branch, Structures Division, Flight Dynamics Directorate, WPAFB, Ohio, with Major Mark Ewing as the technical monitor. The analysis was performed by Jay H. Kim, Assistant Professor, Department of Mechanical Industrial and Nuclear Engineering, University of Cincinnati, Cincinnati, Ohio.



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1. INTRODUCTION

An accurate mathematical model of structures is extremely important for the analysis of modern aerospace or other complex structures. In many cases, the Finite Element Method is practically the only possible method for the analysis of such structures. The accuracy of the finite element analysis depends on how the model of the structure is made. Reliable modelling of complex structures is not possible without refinement of the model compared with experimental results. Especially, damping parameters, structural characteristics of joint, and boundary conditions are very difficult to be estimated, which cause discrepancy between the analytic result and actual measured behavior of the real structure. To overcome such difficulties, numerous schemes of the finite element model update have been developed and reported by many researchers [1]. In general, these methods are to update the analytical model so that it can predict the dynamic response of the structure more closely to the result of modal testing. The underlying assumption is that the test result is more reliable, which is arguable itself. Essentially the task poses an inverse problem because it is an attempt to find the equations of motion from a limited, insufficient set of solutions of the equations. Therefore, none of the methods can provide a unique solution of the problem in all situations. However, a reasonable degree of accuracy in finite element model update/identification is obtainable with proper assumptions and clever strategy.

In the first part of this study, a few representative methods of the structural identification were reviewed and their relative advantages and disadvantages were compared. In the second part, an identification method for the damping properties of the structure has been developed based on the work by Junkins [2]. The main objective of the damping identification is to obtain the damping matrix of the finite element model utilizing measured data. In this study, the main

focus has been put on the identification of the structures with nonproportional viscous damping, which covers the proportional damping as a special case. An algorithm for numerical implementation has been developed and a computer program has been developed for the purpose of method verification during the study. The program can identify the damping matrix of discrete lumped parameter dynamic systems of any degree of freedom and boundary configuration. The method is confirmed to be working satisfactorily for most viscously underdamped structures. Although the program was developed for the spring-mass-damper system, the method can easily be incorporated to the existing identification program for frame structures developed in Wright Laboratory [3,4]. An algorithm to extend the method to identify other types of damping such as structural or Coulomb damping has been suggested.

2. BASIC THEORETICAL BACKGROUND

2.1 Undamped System

Equations of motion of the structure with n degree of freedom are expressed as:

$$[m]\{\ddot{q}\} + [c]\{\dot{q}\} + [k]\{q\} = \{f\} \tag{1}$$

where n by n symmetric matrices [m], [c], and [k] represent inertia, damping and stiffness of the structure which are formulated by finite element method, {q} and {f} are displacement and force vectors of order n. In case of undamped free vibration [c] = [0] and {f} = {0}, by letting: $\{q\} = \{Q\} e^{j\omega t}$,we obtain a classical eigenvalue problem:

$$[k - \omega^2 m] = \{0\} \tag{2}$$

In general, solving equation (2) gives n eigenvalues and eigenvectors which represent natural frequencies and normal modes of the structure. The eigenvectors are mutually orthogonal with respect to [m] and [k] matrices:

$$\{\psi_{\alpha i}^T\}[m]\{\psi_{\alpha j}\} = \delta_{ij}, \quad i,j = 1,2,....,n \tag{3}$$

$$\{\psi_{\alpha i}^T\}[k]\{\psi_{\alpha j}\} = \omega^2_{\alpha i}\delta_{ij}, \quad i,j = 1,2,....,n \tag{4}$$

where $\{\psi_{\alpha i}\}$, $\omega_{\alpha i}$ are the ith undamped natural mode and frequency, and δ_{ij} is the Kronecker delta function. Usually, the frequencies and modes by solving equation (2) are not the same as measured data; therefore, the need for the analytical model update rises. One can view the

identification problem as to find out elements of [m] and [k] matrices of the analytical model so that the resulting frequencies and modes are reasonably close to the exact solution, which are assumed to be the measurement result in general. Since the matrices are symmetric, there are n(n+1) elements to be identified in [m] and [k] matrices if we want to identify all of them. As an ideal case, if all n modes were available from the measurements in compatible coordinate with the analytic model, comparing natural frequencies would give us n equations:

$$\omega_{0i} = \hat{\omega}_{0i}, \quad i = 1,2,3,...,n \tag{5}$$

where $\hat{\omega}_{0i}$ represents the ith measured undamped natural frequency (variables with hats stand for measured quantities hereafter). Comparing n normalized modes would provide n^2 equations:

$$\{\psi_{0i}\} = \{\hat{\psi}_{0i}\}, \quad i=1,2,3,...,n \tag{6}$$

Therefore, theoretically it is possible to identify [m] and [k] matrices uniquely. In reality, there are many difficulties which make this impossible. At first, only a limited number of modes are available, especially in the measured data. Secondly, the experimental model and analytical model may be incompatible in geometry; therefore, a mapping method is required to compare them. Finally, the experimental model itself includes its inherent errors and approximations.

2.2 Damped System

First, let us consider a proportionally damped structure as a special case. By definition, the damping matrix [c] is represented as a linear combination of mass and stiffness matrices.

As in the undamped case, by letting $\{q(t)\} = e^{\lambda t}\{\psi\}$, we obtain an eigenvalue problem:

$$\left[\frac{\lambda(\lambda+\alpha)}{\beta\lambda+1}[m]+[k]\right]\{\psi\}=0 \tag{8}$$

Therefore, if we let $\lambda^* = \frac{\lambda(\lambda+\alpha)}{\beta\lambda+1}$ we obtain an identical form to the undamped equation.

Damped natural frequencies ω_i are obtained because $\lambda_i = j\omega_i$ can be calculated by solving:

$$\lambda^*_i = j\omega_{di} = \frac{\lambda_i(\lambda_i+\alpha)}{(\beta\lambda_i+1)} \quad , i = 1,2,...,n \tag{9}$$

Equation (8) tells that eigenvectors of proportionally damped systems are real and the same as the undamped modes.

For the analysis of the system with nonproportional damping, we have to use an equivalent 2n set of first order ordinary equation of motions known as Hamilton's canonical form [5]:

$$[A]\begin{Bmatrix}\dot{q}(t)\\ \ddot{q}(t)\end{Bmatrix}+[B]\begin{Bmatrix}q(t)\\ \dot{q}(t)\end{Bmatrix}=\{0\} \tag{10}$$

$$\textit{where} \quad A=\begin{bmatrix}-k & 0\\ 0 & m\end{bmatrix}, \quad B=\begin{bmatrix}0 & k\\ k & c\end{bmatrix} \tag{11}$$

Hence , if we let $\begin{Bmatrix}q(t)\\ \dot{q}(t)\end{Bmatrix} = e^{\lambda t}\begin{Bmatrix}\Phi\\ \lambda\Phi\end{Bmatrix} = e^{\lambda t}\{\Phi\}$, The eigenvalue problem becomes:

$$[\lambda A + B]\{\Phi\} = \{0\} \tag{12}$$

Solving equation (12), one can obtain 2n eigenvalues and 2n eigenfunctions, half of them are merely the complex conjugates of the other half. Eigenfunctions have not only arbitrary

magnitudes but also arbitrary phase angles. For relatively light damping, the real and imaginary parts of the eigenvalues can be written in similar form to the proportionally damped case:

$$\lambda_r = -\zeta_r \omega_{or} + j\sqrt{1-\zeta_r^2}\,\omega_{or}, \quad r = 1,2,....,2n \tag{13}$$

where, ζ_r are damping ratios. However, it should be noted that the equation (13) is valid only in an approximate sense for lightly damped structures.

3. REVIEW OF PREVIOUS WORKS

Commonly used criteria to compare the analysis to measurements are natural frequencies, natural modes, and orthogonal conditions of the measured modes with respect to the analytical mass and/or stiffness matrices. The majority of works are involved with minimization of a certain functional which is usually an error norm defined by the difference between analytic and measured results. In some cases, only the orthogonality and/or symmetry of the matrices were used which leads to a set of simultaneous linear equations.

3.1 Approach by Berman and Baruch [6,7]

In their approach, error norms defined by the stiffness and mass matrices are minimized using the orthorgonality conditions, equations of motion and symmetry of the matrices as constraints. For example, if the analytical mass matrix is considered more reliable, the mass matrix is refined first to satisfy the orthogonality condition:

$$\Psi = \left\| M_A^{-\frac{1}{2}} (M - M_A) M_A^{-\frac{1}{2}} \right\| + \sum_i \sum_j \lambda_{ij} (\hat{\psi}_i^T M \hat{\psi}_j - \delta_{ij}) \tag{14}$$

where M_A is the analytic mass matrix to be updated, M is the desired final mass matrix, λ_{ij} are Lagrange multipliers and $\hat{\psi}_i$ are experimental mode shapes. Solving equation (14), the nearest mass matrix from the current analytic mass matrix which satisfies the orthogonality with respect to the measured modes is found. Then, the stiffness matrix [K] is updated to satisfy the symmetry of itself, equations of motion, and orthogonality conditions. That is, minimize:

$$\Psi_2 = \left\| M^{-\frac{1}{2}} (K - K_A) M^{-\frac{1}{2}} \right\| + \sum_i \lambda_i (K \hat{\psi}_i - M \hat{\psi}_i \hat{\omega}_{oi}^2) \tag{15}$$
$$+ \sum_i \sum_j \Lambda_{ij} (\hat{\psi}_i^T K \hat{\psi}_j - \hat{\omega}_{oi} \delta_{ij}) + \sum_i \sum_j \Gamma_{ij} (K_{ij} - K_{ji})$$

If the stiffness matrix is believed to be more reliable, the sequence can be reversed. The problem associated with this method is that it tends to average out errors because there are not enough numbers of constraints to guarantee uniqueness of the solution in most practical cases. For example, the error which is from boundary condition assessment could be spread out to other parameters. Although the resulting equations of motion may produce correct natural frequencies and modes for the given configuration and frequency range, it does not guarantee that the model is accurate for other configurations. Also, the identified matrices become almost fully populated which is not physically valid because the method operates directly on the matrix elements rather than using an update scheme linked to physical properties of the model such as sectional thickness or area. One advantage of this method is that it requires very little computation compared with other methods, because closed form expressions of the [M] and [K] matrices are obtained in terms of $[K_A]$,$[M_A]$, and measured modes.

3.2 Chen and Garba's Method [8]

Chen and Garba's method is to minimize:

$$\Psi = \Delta r_1^2 + \Delta r_2^2 + ... + \Delta r_n^2 \tag{16}$$

with the constraint to satisfy:

$$\Delta f = \left[\frac{\partial f}{\partial r}\right]\{\Delta r\} \tag{17}$$

where $\{\Delta r\}$ represents the necessary update of physical properties and $\{\Delta f\}$ represents the errors in modes and frequencies. Therefore, the method finds a set of the smallest possible update $\{\Delta r\}$ that is compatible with the first order perturbation equations of motion. The fact that the first order perturbation is used in the constraint equation and minimum variation from

the initial analytic model is sought implies that the analytic model is assumed to be reasonably accurate. They also developed a method to calculate the Jacobian matrix used in equation (17) using the perturbation technique so that simultaneous equations are solved instead of the new eigenvalue problem. They used the linked update scheme which updates actual physical parameters to preserve the physical connectivity of the model.

3.3 Approach by Hanagud [9]

S.V. Hanagud also used the first order perturbation equation of the motion to identify the mass, stiffness and damping matrices with a physical parameter update scheme. Since the method is based on the first order perturbation method (as the method by Chen and Garba), the method is considered to be valid only for small error cases. Besides the perturbed equations of motion, no other conditions were used for the identification, which may cause some problems. For example, the updated system matrices may not be orthogonal to the measured modes because no related conditions are used.

3.4 Approach by Junkins and Creamer [2]

Junkins and Creamer used frequency response functions directly for the identification. One can normalize the experimental modes $\hat{\psi}_{ij}$ with respect to the mass matrix by introducing $\phi_{jr} = \sqrt{\alpha_r}\,\hat{\psi}_{jr}$, $r = 1,2,....m$ where ϕ_j are normalized modes. By equating the truncated frequency response functions (equation 18) to the full frequency response function described in terms of normalized mode ϕ_{ij} (equation 19):

$$h_{jk}(\omega) = \sum_{r=1}^{n} \frac{\phi_{jr}\phi_{kr}}{\omega_r^2 - \omega^2} \tag{18}$$

$$\hat{h}_{jk} = \frac{a_1}{\omega^2} + \sum_{r=1}^{m} \left(\frac{\hat{\psi}_{jr} \hat{\psi}_{kr}}{\hat{\omega}_r^2 - \omega^2} \alpha_r \right) + a_2 \tag{19}$$

one can calculate a_1 *and* a_2, which are the effects of rigid body mode and truncated modes, and normalization factors α_r of the measure modes. Then they utilized the orthogonality conditions of the modes to find the mass and stiffness matrices directly from the measured FRF. Because the mode normalization is essentially a part of experimental modal analysis, only the orthogonality conditions were used for the system identification. They proposed to identify the damping matrix in a separate step utilizing the identified mass and stiffness matrix. The first order perturbations from the undamped equations of motion have been used to identify the damping matrix of the structure. This method of damping identification can be used in conjunction with any other undamped structural identification methods. The work in this study for damping identification is based upon this method.

3.5 Identification Using Nonlinear Optimization Technique

Ewing and Venkayya [4], Ewing and Kolonay [3] used a constrained nonlinear optimization method for the structural identification. Which is:

$$\textit{minimize} \quad \left| \sum_{j=1}^{n} (\omega_j - \hat{\omega}_j)^2 \right| \tag{20 a}$$

$$\textit{or} \quad \left| \phi_j - \hat{\phi}_j \right|^2 \tag{20 b}$$

$$\textit{subject to the constraints} \quad \left\| \sum_{j=1}^{n} (\phi_j - \hat{\phi}_j)^2 \right| < \epsilon \tag{21}$$

where ϵ is a small error bound. Any other combinations of equations (20) and (21), and any other constraints such as the total mass of the structure known, may also be used. For the update

of parameters, they used physical parameter update, a similar concept to the macro element approach [10]. There are a few important advantages of ing the nonlinear optimization technique for structural identification. At first, the small error assumption is not necessary unlike all other methods, which could be very useful for the identification of some obscure parameters such as boundary conditions and joint flexibility which might have large errors. Secondly, because the nonlinear optimization theory is very mature and well established, there are many existing state of art utilities readily to be used such as ASTROS [11]. Since it is necessary to compute the sensitivity of the objective function defined by eigenvalue solutions, the method will certainly require a lot more computational effort compared with least square based or other type noniterative type solution. This may be partially alleviated by using Nelson's method for the sensitivity calculation [12,13], although the method is still very computational and may cause another numerical difficulty when the physical and analytical models have incompatible coordinates. However, the computational effort may not be a serious problem for most of practical applications if only a few parameters are to be identified, which are probably the most practical cases. For example, even in complex truss structures, the area and moment of inertia of a typical section are so obvious that they don't need to be or should not be updated. Therefore, there are typically much smaller number of structural properties to be identified, such as flexibility of boundary support and truss joint, or sectional properties of a few really complex and obscure elements. Also, the method can be used with any other method to make a hybrid type identification procedure. For example, a few key parameters can be identified by this method and other parameters are to be fine tuned in the next step by using a least square based method.

4. IDENTIFICATION OF DAMPING MATRIX

Junkins and Creamer[2] used a perturbation technique to identify the damping matrix from the experimental damping ratio and undamped identification results. The damped equations of motion in state space form are given in equation (10). If we consider only the structures with small, viscous damping, one can expand the matrix B in undamped and damped parts:

$$B = \begin{bmatrix} 0 & k \\ k & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & c \end{bmatrix} = B_o + B_1 \tag{22}$$

Let us consider an undamped eigenvalue problem:

$$-\lambda_{or} A_o \Phi_{or} = B_o \Phi_{or}, \quad r = 1,2,....,2n \tag{23}$$

Equation of motion for a damped system can be thought of as a perturbation of equation (10):

$$-\lambda_r A_o \phi_r = B \phi_r, \quad r = 1,2,...,2n \tag{24}$$

$$\textit{where, } \lambda_r = \lambda_{0r} + \lambda_{1r} \quad \textit{and} \quad \Phi_r = \Phi_{0r} + \Phi_{1r}, \quad r = 1,2,....,2n \tag{25}$$

The subscript o in above equations means undamped quantities and 1 means quantities due to the first order perturbation to consider the effect of damping. Expanding the equation (24) and neglecting higher orders, we obtain:

$$B_1 \Phi_{0r} + B_o \Phi_{1r} = -\lambda_{or} A_o \Phi_{1r} - \lambda_{1r} A_o \Phi_{or}, \quad r = 1,2,...,2n \tag{26}$$

The perturbed mode can be expanded as a linear combination of other undamped modes [5]:

$$\Phi_{1r} = \sum_{k=1}^{2n} e_{rk} \Phi_{ok}, \quad \textit{where} \quad e_{rr} = 0 \tag{27}$$

Substituting the equation (27) and utilizing the orthorgonality of the undamped modes, the expression of the perturbed eigenvalue is obtained:

$$\lambda_{1r} = -\Phi_{or}^T B_1 \Phi_{or}, \quad r = 1,2,3,...,2n \tag{28}$$

λ_{1r} is able to be considered as the difference between the measured, damped eigenvalue and undamped eigenvalue:

$$\lambda_{1r} = \hat{\lambda}_r - \lambda_{or} = (\hat{\sigma}_r + j\hat{\omega}_r) - j\omega_{or}, \quad r = 1,2,...., 2n \tag{29}$$

If the undamped normal modes are normalized with respect to the matrix A as in reference [2]:

$$\Phi_{or} = \frac{j}{\sqrt{2}\omega_{or}} \begin{Bmatrix} \phi_{or} \\ j\omega_{or}\phi_{or} \end{Bmatrix} \tag{30}$$

Therefore, by substituting equation (30) to equation (28), we obtain:

$$\sigma_r + j(\omega_r - \omega_{or}) = -\frac{1}{2}\phi_{or}^T \left[(\sum_{q=1}^{R} \xi_q C_q)\phi_{or} \right] \tag{31}$$

where, there are R subgroups of elements with different damping parameters to be identified, ξ_r are parameters to adjust the initial damping estimation C_r of the rth subgroup. For relatively small damping, we can neglect the imaginary part of the left-hand side of the equation (31), which makes the coefficient ξ_q real. Then equation (31) can be arranged in a matrix form:

$$\begin{Bmatrix} -2\sigma_1 \\ . \\ . \\ . \\ -2\sigma_m \end{Bmatrix} = \begin{bmatrix} \phi_{o1}^T C_1 \phi_{o1} & . & . & . & \phi_{o1}^T C_R \phi_{o1} \\ . & & . & & . \\ \phi_{om}^T C_1 \phi om & . & . & . & \phi_{om}^T C_R \Phi_{om} \end{bmatrix} \begin{Bmatrix} \xi_1 \\ \xi_2 \\ . \\ . \\ . \\ \xi_R \end{Bmatrix} \tag{32}$$

The coefficients $\xi_1, \xi_2, \ldots$, in turn the damping matrix, are obtained by solving equation (32). When the number of available modes m is greater than the number of the damping submatrices to be identified, one can use a least square method or the Singular Value Decomposition method

[14]. Some other methods of damping identification can be found in references [15] and [16].

A computer program to implement and verify the procedure described in this section has been developed. The method seems to be working well for a system with relatively small damping, say up to 40% of the critical damping, although it appears to depend on other conditions too. The program can handle a lumped parameter vibration system of any degrees of freedom. The routine can be easily incorporated into any undamped identification algorithm of framed structures.

5. POSSIBLE FUTURE EXTENSION OF THE WORK

5.1 Identification Method in General

Further Research of Identification Methods Using Nonlinear Optimization Technique:

(1) With experimental support, the relative advantage of the method for the case which needs relatively large scale parameter update may be demonstrated. For example, an identification of a few highly uncertain parameters such as unknown boundary conditions can be done by this method with the backup of experimental measurements, and the results can be compared with those from other methods. Also, a hybrid method of the sensitivity based method and other non-iterative methods can be attempted. For example, a few highly uncertain parameters can be identified by the first method, then a method similar to Chen and Garba can be used for all other parameters in the next step.

(2) Another interesting work will be utilizing measurement data of more than one configuration of the structure for the identification purpose [17,18]. For example, a structure may be measured with several radically different boundary conditions, or as divided into subsections. There are a few advantages of doing this although it will require more work in the experimental and identification procedures. First, because any identification scheme is using an incomplete set of information, more experimental information will enhance the reliability of the identification. Second, multiple configurations will provide better experimental quality because it will reduce the possibility to miss certain modes and other types of experimental errors. Last, but perhaps it may be most important, it is equivalently a simulation of the full cycle of actual structural development which includes the modelling, testing, identification, and design change. Therefore, it may provide some idea on how useful the identification work will be when the real structure is developed.

5.2 Extension of the Damping Identification Method to the Structures with Hysteretic and Coulomb Damping

The damping identification method used in this study is to identify relatively lightly, viscously damped structures. The method can be extended to other types of damping such as structural (hysteretic) damping and Coulomb damping. Structural damping is an important subject in the application of aerospace structures because of the increased use of composite materials. In fact any material has structural damping characteristics to some degree, by nature. Since damping parameters of this type are frequency dependent, it is necessary to extend the perturbed equation of the method used in this study. In equation (12), both matrices [A] and [B] become complex coefficient matrices because the stiffness matrix [k] is represented as:

$$[k] = \sum_{i=1}^{R_1} [(1 + j\eta_i)k_i] \tag{33}$$

where η_i represents the structural damping coefficient of the ith element group. Since the matrices A and B in equation (12) are not Hermitian, the resulting eigenfunctions are not mutually orthogonal. Compared with equation (22), we have to consider the perturbations of both matrices [A] and [B], which are $A = A_o + A_1$ *and* $B = B_o + B_1$. The perturbed eigenfunctions still can be expanded in terms of undamped eigenfunctions. With a similar procedure, the equation (28) will become:

$$\lambda_{1r} = -\Phi_{or}^T [\lambda_{or} A_1 + B_1] \Phi_{or} \tag{34}$$

$$\lambda_{1r} = \lambda_r - j\omega_o \tag{35}$$

where λ_r is the rth root of the experimental system equation:

$$s^2[m] + s[c] + [k] = 0 \tag{36}$$

Therefore, the equation (31) will become:

$$\begin{Bmatrix} \lambda_{11} \\ \lambda_{12} \\ . \\ . \\ \lambda_{1m} \end{Bmatrix} = -\frac{1}{2} \begin{bmatrix} \frac{1}{\omega_{o1}} \phi_{o1}^T \Delta K_1 \phi_{01} & . & . \frac{1}{\omega_{o1}} \phi_{o1}^T \Delta K_{R_1} \phi_{o1}, \phi_{01}^T C_1 \phi_{01} & . & . \phi_{o1}^T C_{R_2} \phi_{01} \\ . & . & . & . & . \\ \frac{1}{\omega_{om}} \phi_{om}^T \Delta K_1 \phi_{om} & . & . \frac{1}{\omega_{om}} \phi_{om}^T \Delta K_{R_2} \phi_{om}, \phi_{om}^T C_1 \phi_{om} & . & . \phi_{om}^T C_{R_1} \phi_{om} \end{bmatrix} \begin{Bmatrix} j\eta_1 \\ j\eta_2 \\ . \\ j\eta_{R_1} \\ \xi_1 \\ . \\ \xi_{R_2} \end{Bmatrix} \tag{37}$$

By solving equation (37) using the least square method or Singular Value Decomposition method, one can identify general damping matrices which may be a combination of viscous and structural damping. The above procedure has to be tested using a computer implementation similar to the one in the Appendix. Equation (37) may be divided into two equations of the real part for the viscous damping and imaginary part for the structural damping.

The Coulomb damping is caused by dry friction in structures; therefore, it is dependent on the amplitude of the relative motion between the friction surfaces. The resulting equation of motion becomes nonlinear. An equivalent structural damping can approximate this type of damping reasonably [19]. A measurement setup with a known amplitude of motion at the friction joint may be used to identify an equivalent structural damping parameter.

6. SUMMARY AND CONCLUSION

Various methods of structural identification have been studied and compared in terms of their basic assumptions and advantages. The nonlinear optimization based method is thought to be promising for the identification of structures with highly uncertain structural parameters.

An algorithm has been developed to construct the damping matrix from the experimental data based on the method by Junkins and Creamer. A computer program has been developed for the purpose of verification of this method. Underlying assumptions of the method are that the stiffness and mass matrices have already been identified at the previous step and experimental information on the damping ratios is available. The method is working well for lightly, viscously damped structures which is the most important in practical applications. If the structure is overdamped, the theory cannot be used, which doesn't seem to be a serious limitation because the vibration parameters are not any more significant in overdamped cases. If the damping becomes higher, for example 60% of the critical damping, the method might give poor results. The accuracy of the result appears to depend on the system configuration as well as damping ratios, which needs more study to know the correlations. A suggestion of a possible algorithm has been made to extend the method to the identification of structures with hysteretic damping and Coulomb damping.

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APPENDIX

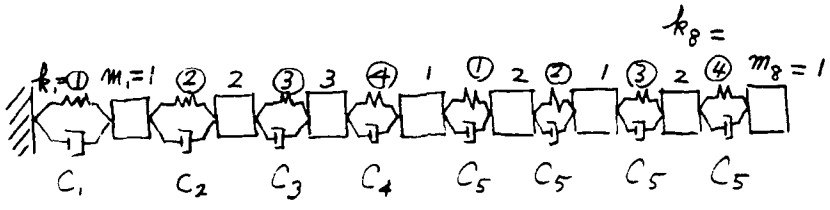
EXAMPLES OF DAMPING MATRIX CALCULATION

```
((( INPUT ECHO )))
ENTER THE FILE NAME YOU WANT TO USE
ESULT.OUT
INPUT OPTION : 1 = PHYSICAL SYSTEM
               2 =  M, C, K DIRECTLY
          1
ENTER THE NUMBER OF NODES
          9
ENTER THE NUMBER OF ELEMENTS
          8
ENTER THE MASS AT THE NODE  1
 0.0000000E+00
ENTER THE MASS AT THE NODE  2
  1.000000
ENTER THE MASS AT THE NODE  3
  2.000000
ENTER THE MASS AT THE NODE  4
  3.000000
ENTER THE MASS AT THE NODE  5
  1.000000
ENTER THE MASS AT THE NODE  6
  2.000000
ENTER THE MASS AT THE NODE  7
  1.000000
ENTER THE MASS AT THE NODE  8
  2.000000
ENTER THE MASS AT THE NODE  9
  1.000000
INFORMATION ON ELEMENT  1
ENTER THE FIRST NODE, SECOND NODE
          1           2
SPRING CONSTANT
  1.000000
INFORMATION ON ELEMENT  2
ENTER THE FIRST NODE, SECOND NODE
          2           3
SPRING CONSTANT
  2.000000
INFORMATION ON ELEMENT  3
ENTER THE FIRST NODE, SECOND NODE
          3           4
SPRING CONSTANT
  3.000000
INFORMATION ON ELEMENT  4
ENTER THE FIRST NODE, SECOND NODE
          4           5
SPRING CONSTANT
  4.000000
INFORMATION ON ELEMENT  5
ENTER THE FIRST NODE, SECOND NODE
          5           6
SPRING CONSTANT
  1.000000
INFORMATION ON ELEMENT  6
ENTER THE FIRST NODE, SECOND NODE
          6           7
SPRING CONSTANT
  2.000000
INFORMATION ON ELEMENT  7
ENTER THE FIRST NODE, SECOND NODE
          7           8
SPRING CONSTANT
  3.000000
INFORMATION ON ELEMENT  8
ENTER THE FIRST NODE, SECOND NODE
          8           9
SPRING CONSTANT
  4.000000
** INFORMATION ON DAMPING **
NO. OF DIFFERENT DAMPING GROUPS
          5
NO. OF ELEMENTS IN THE GROUP  1
          1
MAGNITUDE OF DAMPING
 0.1000000
```

sample input data

```
ENTER ASSOCIATED ELEMENTS  ( 1 ELEMENTS)
          1
NO. OF ELEMENTS IN THE GROUP  2
          1
MAGNITUDE OF DAMPING
 0.2000000
ENTER ASSOCIATED ELEMENTS  ( 1 ELEMENTS)
          2
NO. OF ELEMENTS IN THE GROUP  3
          1
MAGNITUDE OF DAMPING
 0.3000000
ENTER ASSOCIATED ELEMENTS  ( 1 ELEMENTS)
          3
NO. OF ELEMENTS IN THE GROUP  4
          1
MAGNITUDE OF DAMPING
 0.25000000
ENTER ASSOCIATED ELEMENTS  ( 1 ELEMENTS)
          4
NO. OF ELEMENTS IN THE GROUP  5
          4
MAGNITUDE OF DAMPING
 0.5000000
ENTER ASSOCIATED ELEMENTS  ( 4 ELEMENTS)
          5         6         7         8
ENTER THE NODE TO BE FIXED
          1
MORE TO FIX? [Y/N]

NUMBER OF THE MODES TO BE USED FOR DAMPING CALCULATION
          5
```



(System to be identified)

$C_1 = 0.1$

$C_2 = 0.2$

$C_3 = 0.3$

$C_4 = 0.25$

$C_5 = 0.5$

```
6TH EIGENVECTOR
    -0.116193E-03-0.248733E-02   (  0.249005E-02 0.267325E+03 DEG)
     0.184526E-02 0.556889E-02   (  0.586664E-02 0.716673E+02 DEG)
    -0.108272E-01-0.154600E-01   (  0.188743E-01 0.234995E+03 DEG)
     0.616620E-01 0.385349E-01   (  0.727127E-01 0.320028E+02 DEG)
    -0.647483E-01-0.632821E-01   (  0.905371E-01 0.224344E+03 DEG)
     0.166332E+00 0.433229E+00   (  0.464062E+00 0.689965E+02 DEG)
     0.547355E-02-0.332411E+00   (  0.332456E+00 0.270943E+03 DEG)
    -0.804840E-01 0.357053E+00   (  0.366012E+00 0.102703E+03 DEG)
    -0.656216E-02 0.180547E-02   (  0.680600E-02 0.164617E+03 DEG)
     0.137292E-01-0.826056E-02   (  0.160227E-01 0.328966E+03 DEG)
    -0.347049E-01 0.381504E-01   (  0.515741E-01 0.132292E+03 DEG)
     0.656549E-01-0.187494E+00   (  0.198657E+00 0.289299E+03 DEG)
    -0.129770E+00 0.210598E+00   (  0.247369E+00 0.121641E+03 DEG)
     0.105474E+01-0.703602E+00   (  0.126789E+01 0.326293E+03 DEG)
    -0.889264E+00 0.185128E+00   (  0.908329E+00 0.168240E+03 DEG)
     0.100000E+01 0.000000E+00   (  0.100000E+01 0.000000E+00 DEG)

** IDENTIFIED DAMPING **

  GROUP  1  DAMPING VALUE =      0.0956
   ASSOCIATED ELEMENTS
          1

  GROUP  2  DAMPING VALUE =      0.1920
   ASSOCIATED ELEMENTS
          2

  GROUP  3  DAMPING VALUE =      0.3171
   ASSOCIATED ELEMENTS
          3

  GROUP  4  DAMPING VALUE =      0.4832
   ASSOCIATED ELEMENTS
          4

  GROUP  5  DAMPING VALUE =      0.4957
   ASSOCIATED ELEMENTS
          5           6           7           8
```

part of ouput

```
(REFRENCE DATA)
MASS MATRIX

ROW =  1
        1.0000      0.0000      0.0000      0.0000      0.0000      0.0000
   0.0000      0.0000
ROW =  2
        0.0000      2.0000      0.0000      0.0000      0.0000      0.0000
   0.0000      0.0000
ROW =  3
        0.0000      0.0000      3.0000      0.0000      0.0000      0.0000
   0.0000      0.0000
ROW =  4
        0.0000      0.0000      0.0000      1.0000      0.0000      0.0000
   0.0000      0.0000
ROW =  5
        0.0000      0.0000      0.0000      0.0000      2.0000      0.0000
```

Identified Damping Values
and Number of Modes Used

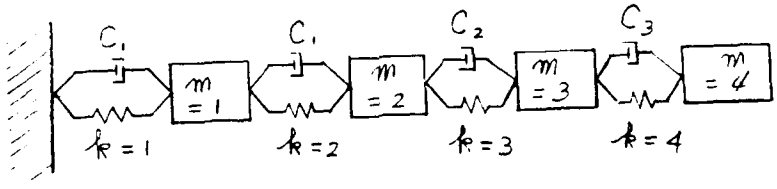
identified damping	number of modes used	
	3	4
C1	0.1	0.1
C2	0.2	0.2
C3	0.3001	0.3
	Equation solver	Least Square Method

*1 : Least Square Method gives as good result as using equation solver

EFFECT OF DAMPING RATIO

	damping values to be found		
	C1	nondimensional damping ratio	Identified values
C1	1.0	0.102	1.0017
C2	2.0	0.421	1.9763
C3	3.0	0.612	3.0522

*1: In this case, identification was made fairly accurately for a system with 60% damping ratio.
*2: Overdamped cases give completely wrong results.



SOME IDENTIFICATION RESULTS:

*1 : Effect of modes used
*2 : Effect of damping magnitude

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